Abstract—Automobile Industry is one of the key industries in the business sector. Automobile industry has the largest growth because of changes in consumer preferences, business policies and changing economic conditions. Almost all modern cars have independent front suspension, which means that each front wheel of your auto is linked separately to the automobile frame. The front wheels must steer as well as respond to the road surfaces because of bounces up and down when either wheel hits a bump or pothole. Automobile should drive down on the road in comfort and safety.

The performance of front suspension of automobile is based on the steering geometry of the suspension. The steering geometry parameters are kingpin angle, camber angle, caster angle, scrub radius, toe in and toe out. In this paper model equations are formulated based on which steering geometry can be predicted and vehicle performance can be determined.

Index Terms—Front Suspension, Steering Geometry, Automobile, Modeling.

I. INTRODUCTION

The Front suspension comprises of a linkage which is a 3 Dimensional mechanism RSSR (Revolute, Spherical, Spherical, Revolute paired). Steering performance depends on the appropriateness of spherical bushes located at various joints. Steering performance depends on the position of kingpin axis. Depending on the position of kingpin axis, kingpin angle, caster angle, camber angle, toe angle and scrub radius of a vehicle are decided.

Presently paper details the steering geometry parameters such as kingpin inclination angle, caster angle, camber angle, toe angle are calculated using model equations derived using experimental data based model theory.

II. FRONT SUSPENSION MECHANISM

A. Working of 3D Front Suspension

On the basis of six included angles of the 3D front suspension mechanism, one at each revolute joints and two at each spherical joints of this four bar chain, position of kingpin axis is determined. Steering performance depends on the position of kingpin axis. Depending on the position of Kingpin axis, Caster angle, Camber angle, Kingpin angle and toe angle of four wheel vehicle are decided. Position of kingpin axis is determined using modeling techniques describe in the paper using dimensionless parameters involved in the experimentation [1].

B. RSSR Mechanism

Joint O₁ and O₂ are revolute joints and joints A and B are Spherical joints as shown in Figure 1. The relative orientation of two links connected at joint can be decided in terms of magnitudes of included angles which in turn can be measured by potentiometer and associated electronic instrumentation. Six potentiometers are located at four joints (two spherical and two revolute) of the RSSR mechanism. At revolute joints O₁ & O₂ the one included angle each of these joints and at spherical joints A & B the two included angles at each of these joints [2].

III. DESIGN OF AN EXPERIMENTAL SET UP

The planning of experimentation is carried out by using the classical plan of experimentation [3]

A. Identification of various Physical Quantities affecting Front Suspension Geometry

The variables affecting the vehicle performance in the context of phenomena of steering are given below.

Independent Variables

1. Length of Upper control arm
2. Length of Lower control arm
3. Length of Knuckle arm
4. Length of Fixed link
5. Diameter of wheel
6. Mass of wheel
7. Road surface roughness in terms of Braker Height
8. Road surface roughness in terms of Braker Width
9. Wheel linear velocity
10. Operational time
11. Acceleration due to gravity
12. Clearance at spherical joint A
13. Clearance at spherical joint B
14. Clearance at revolute joint O1
15. Clearance at revolute joint O2
16. Lateral displacement
17. Spindle length

**Dependent Variables**
1. Kingpin angle
2. Camber angle
3. Caster angle
4. Toe angle
5. Toe in
6. Toe out
7. Scrub radius

**Dimensional Analysis**
The process variables, their symbols and dimensions are listed in Table 1

<table>
<thead>
<tr>
<th>Sr No</th>
<th>Description</th>
<th>Symbol</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Upper control arm</td>
<td>Ua</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>02</td>
<td>Lower control arm</td>
<td>La</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>03</td>
<td>Knuckle arm</td>
<td>Ka</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>04</td>
<td>Fixed link</td>
<td>Fi</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>05</td>
<td>Diameter of wheel</td>
<td>Dw</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>06</td>
<td>Weight of wheel</td>
<td>Wt</td>
<td>( M L^T^3 )</td>
</tr>
<tr>
<td>07</td>
<td>Braker Height</td>
<td>Bh</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>08</td>
<td>Braker Width</td>
<td>Bw</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>09</td>
<td>Wheel velocity</td>
<td>Vt</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>10</td>
<td>Operational time</td>
<td>t</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>11</td>
<td>Acceleration due to gravity</td>
<td>g</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>12</td>
<td>Clearance spherical joint A</td>
<td>Ca</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>13</td>
<td>Clearance spherical joint B</td>
<td>Cb</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>14</td>
<td>Clearance revolute joint O1</td>
<td>Co1</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>15</td>
<td>Clearance revolute joint O2</td>
<td>Co2</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>16</td>
<td>Lateral displacement</td>
<td>Ld</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>17</td>
<td>Spindle length</td>
<td>Sl</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>18</td>
<td>Kingpin angle</td>
<td>Kga</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>19</td>
<td>Camber angle</td>
<td>Cs</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>20</td>
<td>Caster angle</td>
<td>Cm</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>21</td>
<td>Toe angle</td>
<td>Ta</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>22</td>
<td>Toe in</td>
<td>Ti</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>23</td>
<td>Toe out</td>
<td>To</td>
<td>( M^0 L^T^0 )</td>
</tr>
<tr>
<td>24</td>
<td>Scrub radius</td>
<td>Sr</td>
<td>( M^0 L^T^0 )</td>
</tr>
</tbody>
</table>

M, L and T are the symbols for mass, length and time respectively.

**Kingpin angle**
\[ Kga = f(Ua, La, Ka, Fi, Dw, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, SI) \]  
(1)

**Camber angle**
\[ Cs = f(Ua, La, Ka, Fi, Dw, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, SI) \]  
(2)

\[ Cm = f(Ua, La, Ka, Fi, Dw, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, SI) \]  
(3)

\[ Ta = f(Ua, La, Ka, Fi, Dw, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, SI) \]  
(4)

\[ Ti = f(Ua, La, Ka, Fi, Dw, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, SI) \]  
(5)

\[ To = f(Ua, La, Ka, Fi, Dw, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, SI) \]  
(6)

\[ Sr = f(Ua, La, Ka, Fi, Dw, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, SI) \]  
(7)

The process variables, their symbols and dimensions are listed in Table 1.
C. Formulation of Experimental Data Based Model:

Seven independent pi terms ($\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7$) and seven dependent pi terms ($\pi_{D1}, \pi_{D2}, \pi_{D3}, \pi_{D4}, \pi_{D5}, \pi_{D6}, \pi_{D7}$) are decided during experimentation and hence are available for the model formulation.

Each dependent $\pi$ term is the function of the available independent $\pi$ terms,

$$K_{ga} = k_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1} \times (\pi_4)^{d1} \times (\pi_5)^{d1} \times (\pi_6)^{f1} \times (\pi_7)^{g1}$$

The values of exponent’s $a_1, b_1, c_1, d_1, e_1, f_1, g_1$ are established independently at a time, on the basis of data collected through classical experimentation.

Camber angle, $Cm = k_2 \times (\pi_1)^{a2} \times (\pi_2)^{b2} \times (\pi_3)^{c2} \times (\pi_4)^{d2} \times (\pi_5)^{e2} \times (\pi_6)^{f2} \times (\pi_7)^{g2}$

Caster angle, $Cs = k_3 \times (\pi_1)^{a3} \times (\pi_2)^{b3} \times (\pi_3)^{c3} \times (\pi_4)^{d3} \times (\pi_5)^{e3} \times (\pi_6)^{f3} \times (\pi_7)^{g3}$

Toe angle, $Ta = k_4 \times (\pi_1)^{a4} \times (\pi_2)^{b4} \times (\pi_3)^{c4} \times (\pi_4)^{d4} \times (\pi_5)^{e4} \times (\pi_6)^{f4} \times (\pi_7)^{g4}$

Scrub radius, $Sr = k_5 \times (\pi_1)^{a5} \times (\pi_2)^{b5} \times (\pi_3)^{c5} \times (\pi_4)^{d5} \times (\pi_5)^{e5} \times (\pi_6)^{f5} \times (\pi_7)^{g5}$

Base radius, $Base = k_6 \times (\pi_1)^{a6} \times (\pi_2)^{b6} \times (\pi_3)^{c6} \times (\pi_4)^{d6} \times (\pi_5)^{e6} \times (\pi_6)^{f6} \times (\pi_7)^{g6}$

Base in, $Base_{in} = k_7 \times (\pi_1)^{a7} \times (\pi_2)^{b7} \times (\pi_3)^{c7} \times (\pi_4)^{d7} \times (\pi_5)^{e7} \times (\pi_6)^{f7} \times (\pi_7)^{g7}$

Base out, $Base_{out} = k_8 \times (\pi_1)^{a8} \times (\pi_2)^{b8} \times (\pi_3)^{c8} \times (\pi_4)^{d8} \times (\pi_5)^{e8} \times (\pi_6)^{f8} \times (\pi_7)^{g8}$

From these models values of all dependent pi terms are computed.

There are eight unknown terms in the equation 8 curve fitting constant $K_{ga}$ and indices $a_1, b_1, c_1, d_1, e_1, f_1, g_1$ to get the values of these unknowns we need minimum a set of eight values of ($\pi_{D1}, \pi_{D2}, \pi_{D3}, \pi_{D4}, \pi_{D5}, \pi_{D6}, \pi_{D7}$).

$$Z = a + bx + cy + dz + \ldots$$

The equation 8 can be brought in the form of equation 9 by taking log on both sides.

D. Model Formulation:

Model of dependent pi term $\pi_{D1}$ for kingpin angle

$$K_{ga} = k_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1} \times (\pi_4)^{d1} \times (\pi_5)^{e1} \times (\pi_6)^{f1} \times (\pi_7)^{g1}$$

Taking log on the both sides of equation for $\pi_{D1}$, getting eight unknown terms in the equations,

$$\log \pi_{D1} = \log k_1 + a_1 \log \pi_1 + b_1 \log \pi_2 + c_1 \log \pi_3 + d_1 \log \pi_4 + e_1 \log \pi_5 + f_1 \log \pi_6 + g_1 \log \pi_7$$

Let,

$$Z_1 = \log \pi_{D1}, K_1 = \log k_1, \ A = \log \pi_1, \ B = \log \pi_2, \ C = \log \pi_3, \ \ D = \log \pi_4, \ \ E = \log \pi_5, \ \ F = \log \pi_6, \ \ G = \log \pi_7$$

Putting the values in equations 3, the same can be written as

$$Z_1 = K_1 + A + B + C + D + E + F + G + G$$

Equation 10 is a regression equation of $Z$ on $A, B, C, D, E, F, G$ in an n dimensional co-ordinate system.

$$\Sigma Z_1 = n K_1 + a_1 \Sigma A + b_1 \Sigma B + c_1 \Sigma C + d_1 \Sigma D + e_1 \Sigma E + f_1 \Sigma F + g_1 \Sigma G$$

Putting the values in equations 11 one will get a set of 8 equations, which are to be solved simultaneously to get the values of $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and $g_1$.

There are two unknown terms the values of $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and $g_1$ can be obtained by using matrix analysis.

$$X_1 = \text{inv} (W) \times P_1$$

The matrix method of solving these equations using ‘MATLAB’ is given below.

$$W = 8 \times 8 \text{ matrix multipliers of } K_1, a_1, b_1, c_1, d_1, e_1, f_1, g_1$$

$$P_1 = 8 \times 1 \text{ matrix of the terms on } L H S$$

$$X_1 = 8 \times 1 \text{ matrix of values of } K_1, a_1, b_1, c_1, d_1, e_1, f_1, g_1$$

Then, The matrix obtained is given by,
The models have been formulated mathematically. An approximate generalized experimental data based models are evolved for predicting the Steering Behavior [7].

This includes application of Dimensional Analysis is quite simple way in which a given test can be made compact in operating plan. In this experimentation we may not be able to recognize all the variables that influence a test, but we should realize that they and their dimensional equation have reality whether or not it is apparent.

The indices of the model are the indicators of how the phenomenon is getting affected because of the interaction of various independent pi terms in the models. The influence of indices of the various independent pi terms on each dependent pi term is discussed below.

The model for the dependent pi term $\pi_{D1}$ in the under $\pi_{D1} = 3.597 \times 10^{29} \pi_{1}^{45.898} \pi_{2}^{52.468} \pi_{3}^{0.080123} \pi_{4}^{11.161}$

It can be seen from the equation that this model of pi terms containing kingpin angle as response variable [6]. The following primary conclusions appear to be justified from the above model.

1. The absolute index of $\pi_1$ is highest viz. 45.898. Thus in $\pi_1$ the terms related to the front suspension link lengths of suspension mechanism are the most influencing factors in this phenomenon. The value of this index is positive indicating $\pi_{D1}$ is directly varying with respect to $\pi_1$.

2. The absolute index of $\pi_2$ is lowest viz. -52.468, then $\pi_2$ term related to wheel diameter is the least influencing pi term in the model. The value of the index is negative indicating $\pi_{D1}$ is inversely varying with respect to $\pi_2$.

3. The sequence of influence of the other independent pi terms present in the model is $\pi_{D1}, \pi_{3}, \pi_{4}, \pi_{5}, \pi_{6}, \pi_{7}$ having absolute indices 3.1647, 0.080123, 0.011445, -0.074352, -0.116116 respectively. The index of $\pi_3$ negative indicating that $\pi_{D1}$ inversely proportional with respect to $\pi_3$ and $\pi_4$.

4. The curve fitting constant in the model is 3.597 x 10^{29}. This value represents the effect of clearances and other factors which affect the phenomena.

### TABLE IV: ERROR OBTAINED FROM EXPERIMENTAL CALCULATED VALUE AND MATHEMATICAL MODEL VALUE FOR KINGPIN ANGLE

<table>
<thead>
<tr>
<th>Kingpin angle</th>
<th>Error Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimentation</td>
<td>Mathematical Model</td>
</tr>
<tr>
<td>6.3154</td>
<td>6.0180</td>
</tr>
<tr>
<td>6.6566</td>
<td>6.4047</td>
</tr>
<tr>
<td>6.8082</td>
<td>6.6689</td>
</tr>
<tr>
<td>6.9931</td>
<td>7.1868</td>
</tr>
<tr>
<td>7.2404</td>
<td>7.3929</td>
</tr>
</tbody>
</table>

V. CONCLUSION

The models values of the dependent pi terms are computed. The observed and computed values of the dependent pi terms are compared by calculating their mean values. In order to check the accuracy of the predicted / computed values of the dependent pi terms, error is worked out(13).

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REFERENCES


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